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B.Com – 4th Semester Business Statistics – II

UNIT – 5 : TRANSPORTATION PROBLEM

Introduction:

In the application of linear programming techniques, the transportation problem was probably one of the first significant problems studied. When a large number of variables (more than 2) are involved in a problem, the solution by graphical method is not possible. The simplex method provides an efficient technique which can be applied for solving linear programming problems of any magnitude, involving two or more decision variables. All linear programming problems can be solved by simplex method, but certain special problems lend themselves to easy solution by other methods. One of such case is "Transportation Problem".

Transportation problems are encountered in physical distribution of goods. Source of supply, availability of material or commodity for distribution, the requirement of demand at particular place or destination or at number of destinations are some of the parameters involved in the problem. The objective is to minimise the cost associated with such transportation from place of supply to places of demand within given constraints of availability and level of demand. These distribution problems are amenable to solution by a special type of linear programming model known as "Transportation Model". It can also be applied to the maximisation of some utility value such as financial resources.

Meaning and Definition:

Transportation model is a special type of networks problems dealing with shipping a commodity from source (e.g., factories) to destinations (e.g., warehouse). It deals with obtaining the minimum cost plan to transport a commodity from a number of sources (**m**) to number of destination (**n**). The objective of transportation problem is to determine the amount to be transported from each origin to each destination such that the total transportation cost is minimized.

Structure of the Transportation Problem (TP):

Let, m = Origin n = Destinations

a_i = Quantity of products available at origin i

b_j = Quantity of products required at destination j

C_{ij} = Cost of transporting one unit of product from origin i to destination j

x_{ij} = Quantity transported from origin i to destination j

The objective is to determine the quantity x_{ij} to be transported over all routes (i, j) so as to minimize the total transportation cost. The quantity of products available at the origins must satisfy the quantity of products required at the destinations.

If this is represented in matrix form, wherein each cell contains a transportation variable, means the number of units shipped from the row designated origin to the column designated destination. The amount of supplies a_i available at source i and the amount demanded by b_j at each destination j. Hence, a_i and b_j represent supply and demand constraint.

This transportation problem can be represented through matrix as follows:

To From		Destination					
		D_1	D_2	D_3	. . . . .	D_n	Supply
Origin	S_1	(x_{11}) C_{11}	(x_{12}) C_{12}	(x_{13}) C_{13}	. . . . .	(x_{1n}) C_{1n}	a_1
	S_2	(x_{21}) C_{21}	(x_{22}) C_{22}	(x_{23}) C_{23}	. . . . .	(x_{2n}) C_{2n}	a_2
	S_3	(x_{31}) C_{31}	(x_{32}) C_{32}	(x_{33}) C_{33}	. . . . .	(x_{3n}) C_{3n}	a_3
	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
	S_m	(x_{m1}) C_{m1}	(x_{m2}) C_{m2}	(x_{m3}) C_{m3}	. . . . .	(x_{mn}) C_{mn}	a_m
	Demand	b_1	b_2	b_3		b_n	$\sum a_i = \sum b_j$

Here S=Source and D=Destination

This represent the case, where total supply = total demand.

i.e. $\sum_{i=1}^{m} ai = \sum_{j=1}^{n} bj$

It is the case when demand is fully met from the origin and the problem can be stated as LP problem in the following manner:

The objective function:

Minimize $Z = \sum_{i=0}^{m} \sum_{j=0}^{n} Cij\ Xij$ subject to the constraints

$\sum_{j=1}^{n} Xij = ai$ for i = 1, 2, 3 . . . . m (Supply constraint)

$\sum_{i=1}^{m} Xij = bj$ for j = 1, 2, 3 . . . . n (Demand constraint)

and $X_{ij} \geq 0$ for all i = 1, 2, 3 . . . . m & j = 1, 2, 3 . . . . n (Non negative constraint)

Balanced transportation problem:

A transportation problem is said to be balanced if the supply from all the sources are equal to the total demand of all destinations. It means,

Total quantity of supply = Total quantity of demand.

Symbolically, $\sum_{i=1}^{m} ai = \sum_{j=1}^{n} bj$

Unbalanced transportation problem:

A transportation problem is said to be unbalanced if the supply from all the sources are not equal to the total demand of all destinations. It means,

Total quantity of supply < OR > Total quantity of demand.

Symbolically, $\sum_{i=1}^{m} ai < \sum_{j=1}^{n} bj$ OR $\sum_{i=1}^{m} ai > \sum_{j=1}^{n} bj$

In case of unbalance transportation problem, first we need to convert this into balanced transportation problem to solve it.

Feasible Solution:

A feasible solution to a transportation problem is a set of non-negative values X_{ij} (i=1,2,......,m and j=1,2,......,n) that satisfies the constraints.

Basic Feasible Solution:

A feasible solution is called a basic feasible solution if it contains not more than m+n-1 allocations, where m is the number of rows and n is the number of columns in a transportation problem.

Optimum Solution:

Optimum solution is a feasible solution (not necessarily basic) which optimizes (minimize) the total transportation cost.

Degeneracy:

If a basic feasible solution to a transportation problem contains less than m+n-1 allocations, it is called a degenerate basic feasible solution and it cannot be tested for optimality. Here m is the number of rows and n is the number of columns in a transportation problem.

Non degenerate basic feasible solution:

If a basic feasible solution to a transportation problem contains exactly m+n-1 allocations in independent positions, it is called a non degenerate basic feasible solution. Here m is the number of rows and n is the number of columns in a transportation problem.

Methods of finding initial Basic Feasible Solutions:

There are several methods available to obtain an initial basic feasible solution of a transportation problem. In order to find the initial basic feasible solution, total supply must be equal to total demand. i.e. $\sum_{i=1}^{m} ai = \sum_{j=1}^{n} bj$. Here, we discuss only the following three methods.

1. North-West Corner Rule or DENTZY's Method
2. Least Cost Method (LCM) or Minimum Cost Method
3. Vogel's Approximation Method (VAM)

North-West Corner Rule or DENTZY's Method:

Initial basic feasible solution can be obtained by following the given below steps:

Step 1: Choose the cell in the north-west corner of the transportation problem table and allocate as much as possible in this cell so that either the capacity of first row (supply) is exhausted or the destination requirement of the first column (demand) is exhausted. (i.e) x_{11} = min (a_1, b_1)

Step 2: If the demand is exhausted ($b_1 < a_1$), move one cell right horizontally to the second column and allocate as much as possible. (i.e) x_{12} = min ($a_1 - x_{11}$, b_2).

If the supply is exhausted ($b_1 > a_1$), move one cell down vertically to the second row and allocate as much as possible. (i.e) x_{21} = min (a_2, $b_1 - x_{11}$).

If both supply and demand are exhausted move one cell diagonally and allocate as much as possible.

Step 3: Continue the above procedure until all the allocations are made.

In this way, move horizontally until a supply source is exhausted, then vertically down until a destination demand is satisfied and diagonally, when the demand at the destination matches exactly the supply available, until the south east corner is reached.

Illustration 01:

Find the initial basic feasible solution for the following transportation problem, using North-West Corner Rule method.

		Destination		
Sources↓	**D1**	**D2**	**D3**	**Supply**
S1	3	8	5	**7**
S2	4	4	2	**8**
S3	6	5	8	**10**
S4	2	6	3	**15**
Demand	**8**	**10**	**22**	

Solution:

Let's compute the total of supply and total of demand

Total Supply = 7 + 8 + 10 + 15 = 40

Total Demand = 8 + 10 + 22 = 40

Since, the total supply = total demand, i.e. $\sum_{i=1}^{m}$ ai = $\sum_{j=1}^{n}$ bj, the problem is said to be balanced TP. Now, we can move ahead with finding an initial basic feasible solution, using the North West corner rule method:

Let's start with first cell (S1,D1) and allocate as much as possible in this cell so that either the capacity of first row is exhausted or the destination requirement of the first column is exhausted. i.e. x_{11} = min (7, 8) = 7

Now, move on to the next cell (S2,D1) and allocate as much as possible in this cell so that either the capacity of second row is exhausted or the destination requirement of the first column is exhausted. i.e. x_{12} = min (1, 8) = 1

Now, move on to the next cell (S2,D2) and allocate as much as possible in this cell so that either the capacity of second row is exhausted or the destination requirement of the second column is exhausted. i.e. x_{22} = min (7, 10) = 7

These steps are to be repeated till we reach south east corner, where entire supply is exhausted with demand of destinations are fulfilled.

After completing entire process, we find the following allocations.

		Destination		
Sources↓	**D1**	**D2**	**D3**	**Supply**
S1	3**(7)**	8	5	**7/0**
S2	4**(1)**	4**(7)**	2	**8/7/0**
S3	6	5**(3)**	8**(7)**	**10/7**
S4	2	6	3**(15)**	**15/0**
Demand	**8/1/0**	**10/3**	**22/15/0**	

Now we have the following transportation schedule:

S1 –> D1   S2 –> D1   S2 –> D2   S3 –> D2   S3 –> D3   S4 –> D3

Therefore Transportation Cost:

(3 * 7) + (4 * 1) + (4 * 7) + (5 * 3) + (8 * 7) + (3 * 15) = **Rs. 169**

Feasibility of the solution: To check the feasibility we need to test allocated cells must be equal to m + n -1. Therefore allocated cells = 6, which is equal to m+n-1 = 4+3-1 = 6. Hence the solution is feasible one.

Illustration 02:

Find out the minimum cost solution for the following transportation problem, using North West Corner Rule method.

To / From	P	Q	R	Availability
A	16	19	12	14
B	22	13	19	16
C	14	28	8	12
Requirement	10	15	17	

Solution:

Total Availability = Total Requirement = 42, hence this is balanced TP. Starting with north west corner cell, initial requirement of 10 for destination P has been met with Source A. Since availability source A is 14, we move horizontally to destination Q to allocate balance of 4 i.e. (14-10). This exhausts the availability at source A and now we move vertically to source B to fulfil the balance requirement of destination Q 11 i.e. (15-4). Now, availability at source B is 5 and requirement at destination Q is exhausted and we move horizontally to next destination R. Here, we allocate 5 i.e. (16-11), which is available from source B. This exhausts the availability at source B and now we move vertically to the next source C to fulfil the balance requirement of destination R 12 i.e.

(17-5). Here, we can see that it fulfils both requirement of destination R and also exhausts the availability at source C which is equal to 12.

To / From	P	Q	R	Availability
A	16**(10)**	19**(4)**	12	14
B	22	13**(11)**	19**(5)**	16
C	14	28	8**(12)**	12
Requirement	10	15	17	

Now we have the following transportation schedule:

A –> P   A –> Q   B –> Q   B –> R   C –> R

Therefore Transportation Cost:

(16 * 10) + (19 * 4) + (13 * 11) + (19 * 5) + (8 * 12) = **Rs. 570**

Feasibility of the solution: In the given problem m + n – 1 = 3 + 3 – 1 = 5. Allocated cells are also equal to 5 and hence the solution is feasible one.

Least Cost Method (LCM) or Minimum Cost Method:

This method is more economical as compared to north west corner rule method, since it considers the lowest cost for allocation from beginning itself. When objective is to minimize cost then it is always better to go through lowest cost approach. Hence, the various steps involved in this method are as follows:

Step 1: Find the cell with the least (minimum) cost in the transportation table.

Step 2: Allocate the maximum feasible quantity to this cell considering the supply as well as demand.

Step 3: Eliminate the row or column where an allocation is made.

Step 4: Adjust the demand or supply for the remaining values after the above allocation.

Step 5: Repeat the above steps for the reduced transportation table until all the allocations are made.

Illustration 03:

Obtain the initial basic feasible solution to the following TP using least cost method.

	D1	D2	D3	D4	Supply
O1	1	2	3	4	6
O2	4	3	2	5	8
O3	5	2	2	1	10
Demand	4	6	8	6	

Here Oi and Dj denote i th origin and j th destination respectively.

Solution:

Total Supply = Total Demand = 24. Therefore, the given problem is a balanced transportation problem. Hence there exists a feasible solution to the given problem.

Given Transportation Problem is:

	$D1$	$D2$	$D3$	$D4$	Supply
$O1$	1	2	3	4	6
$O2$	4	3	2	5	8
$O3$	5	2	2	1	10
Demand	4	6	8	6	

The least cost is 1 corresponds to the cells ($O1$, $D1$) and ($O3$, $D4$)

Take the Cell ($O1$, $D1$) randomly and allocate min (6,4) = 4 units to this cell.

	$D1$	$D2$	$D3$	$D4$	Supply
$O1$	1**(4)**	2	3	4	6/2
$O2$	4	3	2	5	8
$O3$	5	2	2	1	10
Demand	4/0	6	8	6	

The reduced table is:

	$D2$	$D3$	$D4$	Supply
$O1$	2	3	4	2
$O2$	3	2	5	8
$O3$	2	2	1**(6)**	10/4
Demand	6	8	6/0	

The least cost corresponds to the cell (O3, D4). Allocate min (10,6) = 6 units to this cell. Now the reduced table is:

	$D2$	$D3$	Supply
$O1$	2	3	2
$O2$	3	2**(8)**	8/0
$O3$	2	2	4
Demand	6	8/0	

The least cost in the above matrix is 2, which corresponds to the cells ($O1$, $D2$), ($O2$, $D3$), ($O3$, $D2$), ($O3$, $D3$). Choose any cell arbitrarily and allocate min of supply or demand. We will take ($O2$, $D3$) since both demand and supply is same. So that we can eliminate both row as well as column simultaneously.

Now, the reduced table is:

	$D2$	Supply
;$O1$	2**(2)**	2
$O3$	2**(4)**	4
Demand	6	

Now balance units can be allocated to the remaining cell.

Thus we have the following allocations:

	$D1$	$D2$	$D3$	$D4$	Supply
$O1$	1**(4)**	2**(2)**	3	4	6/2/0
$O2$	4	3	2**(8)**	5	8/0
$O3$	5	2**(4)**	2	1**(6)**	10/4/0
Demand	4/0	6/4	8/0	6/0	

Transportation schedule:

$O_1 \rightarrow D_1$ $O_1 \rightarrow D_2$ $O_2 \rightarrow D_3$ $O_3 \rightarrow D_2$ $O_3 \rightarrow D_4$

Total transportation cost

= $(4\times1) + (2\times2) + (8\times2) + (4\times2) + (6\times1)$

= $4 + 4 + 16 + 8 + 6$

= **Rs. 38/-**

Illustration 04:

Factory \ Store	A	B	C	Availability
P	10	8	8	8
Q	10	7	10	7
R	11	9	7	9
S	12	14	10	4
Requirement	10	10	8	

Solution:

Total Availability = Total Requirement = 42, hence this is balanced TP. Now start with allocation to the least cost cell in the matrix. i.e. cost is 7 in the cell (Q,B) as well as (R,C). Now choose any one randomly, let's take (Q,B) and allocate min (10,7) = 7 to the cell (Q,B). Now eliminate the row Q and then find least cost among remaining values i.e. 7 in the cell (R,C). Now allocate min (8,9) = 8 to the cell (R,C) and eliminate column C. Like this proceed further until all supply exhausts and demand is fulfilled.

Store / Factory	A	B	C	Availability	
P	10(5)	8(3)	8	8/5/0	Fourth row elimination
Q	10	7(10)	10	7/0	First row elimination
R	11(1)	9	7(8)	9/1/0	Fifth row elimination
S	12(4)	14	10	4	Last row elimination
Requirement	10/5/4	10/3	8/0		
	Last column elimination	Third column elimination	Second column elimination		

Now we have the following transportation schedule:

P –> A   P –> B   Q –> B   R –> A   R –> C   S –> A

Therefore Transportation Cost:

(10 * 5) + (8 * 3) + (7 * 10) + (11 * 1) + (7 * 8) (12 * 4) = **Rs. 259**

Feasibility of the solution: In the given problem m + n – 1 = 4 + 3 – 1 = 6. Allocated cells are also equal to 6 and hence the solution is feasible one.

Vogel's Approximation Method (VAM) or Penalty Method:

This is a preferred method over earlier two methods due to its solution being either optimal or very near to optimal. This method yields an initial basic feasible solution may reduce the time for optimal calculations. Various steps involved in this method are summarised as under:

Step 1: Consider each row of the cost matrix individually and find the difference between two least cost cells (which is called as penalty) in it.

Step 2: Similarly, repeat the above method to each column also.

Step 3: Identify the column or row with the largest value difference. In case of tie, select any one. But it is wise to select the row or column to give allocation at minimum cost cell).

Step 4: Now, consider the cell with the minimum cost in the column (or row, as the case may be) and assign the maximum units possible, considering the demand and supply positions corresponding to that cell. Assign only one cell at a time.

Step 5: Delete the column or row, where all the allocations are made.

Step 6: Repeat the procedure until all the allocations are made.

The Vogel's approximation method is also known as Penalty Method because the cost differences that it uses are nothing but the penalties of not using the least cost route. Since the objective function is the minimisation of the transportation cost in each iteration route selected involves the maximum penalty of not being used. Under this method computation takes too much time and hence it is slow but due to its advantages, it has been widely used.

Illustration 05:

Find the initial basic feasible solution for the following TP by VAM:

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Requirement	200	225	275	250	

Solution:

Here, Total Availability = Total Requirement, i.e. $\sum_{i=1}^{m}$ ai = $\sum_{j=1}^{n}$ bj = 950, the problem is said to be balanced TP and hence there exist a basic feasible solution. Now, we can move ahead with finding an initial basic feasible solution, using VAM.

First let us find the difference (penalty) between the first two smallest costs in each row and column and write them in brackets against the respective rows and columns

	D1	D2	D3	D4	Supply	Penalty
S1	11**(200)**	13	17	14	250/50	2
S2	16	18	14	10	300	4
S3	21	24	13	10	400	3
Requirement	200/0	225	275	250		
Penalty	5	5	1	4		

Choose the largest difference. Here the difference is 5 which corresponds to column $D1$ and $D2$. Choose either $D1$ or $D2$ arbitrarily. Here we take the column $D1$. In this column choose the least cost. Here the least cost corresponds to ($S1$, $D1$). Allocate min (250, 200) = 200 units to this Cell.

The reduced transportation table is

	D2	D3	D4	Supply	Penalty
S1	13**(50)**	17	14	50/0	1
S2	18	14	10	300	4
S3	24	13	10	400	3
Requirement	225/175	275	250		
Penalty	5	1	4		

Choose the largest difference. Here the difference is 5 which correspond to column D2. In this column choose the least cost. Here the least cost corresponds to (S1, D2). Allocate min (50, 175) = 50 units to this Cell.

The reduced transportation table is

	D2	D3	D4	Supply	Penalty
S2	18**(175)**	14	10	300/125	4
S3	24	13	10	400	3
Requirement	175/0	275	250		
Penalty	6	1	0		

Choose the largest difference. Here the difference is 6 which correspond to column D2. In this column choose the least cost. Here the least cost corresponds to (S2, D2). Allocate min (300, 175) = 175 units to this cell.

The reduced transportation table is

	D3	D4	Supply	Penalty
S2	14	10**(125)**	125/0	4
S3	13	10	400	3
Requirement	275	250/125		
Penalty	1	0		

Choose the largest difference. Here the difference is 4 corresponds to row S2. In this row choose the least cost. Here the least cost corresponds to (S2, D4). Allocate min (125, 250) = 125 units to this Cell.

The reduced transportation table is

	D3	D4	Supply	Penalty
S3	13**(275)**	10**(125)**	400	3
Requirement	275	125		
Penalty	-	-		

Allocate the balance to remaining cells.

Thus we have the following allocations:

	D1	D2	D3	D4	Supply
S1	11**(200)**	13**(50)**	17	14	250/50
S2	16	18**(175)**	14	10**(125)**	300
S3	21	24	13**(275)**	10**(125)**	400
Requirement	200/0	225	275	250	

Now we have the following transportation schedule:

S1 –> D1 S1 –> D2 S2 –> D2 S2 –> D4 S3 –> D3 S3 –> D4

Therefore Transportation Cost:

(11*200) + (13*50) + (18*175) + (10*125) + (13*275) (10*125) = **Rs. 12,075/-**

Feasibility of the solution: In the given problem m + n – 1 = 3 + 4 – 1 = 6. Allocated cells are also equal to 6 and hence the solution is feasible one.

Illustration 06:

Find the initial basic feasible solution for the following TP by VAM:

To / From	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	**150**

Solution:

Here, Total Supply = Total Demand, i.e. $\sum_{i=1}^{m}$ ai = $\sum_{j=1}^{n}$ bj = 150, the problem is said to be balanced TP and hence there exist a basic feasible solution. Now, we can move ahead with finding an initial basic feasible solution, using VAM.

Row A has minimum element as 1 and next least as 4, the difference 4-1=3 is written against iteration-I in row A. Similarly, for row B, the difference of least cost cells is 7-3=4 and is so indicated under iteration-I, in row B. This process is continued for row C and all the columns. Maximum values in row and column difference i.e. 3, 4, 2, 1, 0, 1 is 4 and hence allocation (max=20) is made to the cell of least cost i.e. cell (B,D). This satisfies column D and is scored out. Repeating same process during Iteration-II, we allocate 35 units at cell A,F (cell with least cost in row A). This satisfies column F fully and hence column F is scored out, due to its demand having been met fully. Other allocations are made based on supply/demand positions.

The solution of the TP using VAM is as follows:

To / From		D	E	F	Supply	Iteration (Penalty)	
						I	II
A		6	4**(15)**	1**(35)**	50	3	3
B		3**(20)**	8**(20)**	7	40	4	1
C		4	4**(60)**	2	60	2	2
Demand		20	95	35	**150**		
Iteration (Penalty)	I	1	0	1			
	II	-	0	1			

Now we have the following transportation schedule:

A –> E A –> F B –> D B –> E C –> E

Therefore Transportation Cost:

(4 * 15) + (1 * 35) + (3 * 20) + (8 * 20) + (4 * 60) = **Rs. 555/-**

Feasibility of the solution: In the given problem m + n – 1 = 3 + 3 – 1 = 5. Allocated cells are also equal to 5 and hence the solution is feasible one.

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